On the Developement of Exponential Functions; together with several new Theorems relating to finite Differences. By John Frederick W. Herschel, *Esq. F.R.S.* Read December 14, 1815. [*Phil. Trans.* 1816, *p.* 25.]

The subject here considered by Mr. Herschel relates to the celebrated theorems of Lagrange, expressing the connection between simple exponential indices and those of differentiation and of integration.

Since the theorems have been demonstrated by various subsequent analysts, as by Laplace, by Arbogast, and by Dr. Brinkley, the author takes them for granted; but observes that in their original form they are but abridged expressions of their meaning; and that in order to become practically useful, their exponential functions require further development.

And though this part of the subject has been treated with great ability by Dr. Brinkley, who has deduced formulæ respecting the first of the two theorems far more simple than could have been expected from the complex nature of the subject; yet since his method, when applied to the second more general theorem, would lead to details of extreme complexity, Mr. Herschel has taken a different view of the subject; and beginning with the more general theorem has arrived at a general formula, which he believes to have been hitherto wholly unnoticed, and which, when applied to certain particular cases treated of by Dr. Brinkley, affords precisely the same results.

But the mode in which this subject is treated, or even the results, were not of a nature to admit of being read in public.

On new Properties of Heat, as exhibited in its Propagation along Plates of Glass. By David Brewster, *LL.D. F.R.S. Lond. and Edinb. In a Letter addressed to the Right Hon. Sir* Joseph Banks, *Bart. G.C.B. P.R.S.* Read January 11, 1816. [*Phil. Trans.* 1816, *p.* 46.]

In a paper communicated to the Society in May 1814, Dr. Brewster observed that glass, when raised to a high temperature, had the property of depolarizing light, and in this respect resembled crystallized substances; but he did not at that time succeed in tracing a resemblance in other points, which he left for future investigation.

On resuming this inquiry in the present paper, the subject is divided into two parts; in the former of which he describes the transient effects exhibited during the propagation of heat along plates of glass, whether received from adjacent bodies or communicated to them; and in the latter he describes the permanent optical properties produced in glass by being suddenly and partially cooled when red hot.

The phenomena here noticed depend on the total or partial depolarization of light, previously polarized by reflection at a certain angle from a surface of black glass. It is well known that when a ray of light thus polarized in the plane of primitive incidence is incident upon a second surface of black glass at the same specific angle,

if the second plane of incidence be at right angles to the former, then no part of that ray will be reflected by the second surface; and an eye rightly placed for witnessing the effects, will perceive in the exact specific direction a central spot of absolute blackness, surrounded also by a dark space of some extent, from which less or more of light is reflected, in proportion to the increase of distance from the central line of no reflection. The light, however, which has thus been polarized, may be wholly, or in part, depolarized by the interposition of many crystallized bodies, the degree of depolarization being dependent on the more or less exact position of a certain neutral axis or plane of their crystalline texture with the plane of primitive polarization. When these planes are perfectly coincident, the light remains polarized in that plane, and a black line appears in that direction; but adjacent to it, on each side, are seen a series of colours, which depend partly upon their proximity to the central black line, and partly upon the thickness of the depolarizing body, the succession of colours being exactly the same as those observed by Newton in thin plates, but variously modified in their forms, according to the nature of the crystalline substance interposed, and according to the position of its axis.

The optical effects of heated glass, as now observed by Dr. Brewster, are precisely of the same kind, and are now found to depend not upon the simple circumstance of temperature of the entire plate, as he originally supposed, but upon the progressive differences of temperature in different parts of the plate, arising from contact or proximity to a plate of hot or cold iron, or from the cooling power of the surrounding atmosphere. And in the same manner as the several tints of colour produced by crystallized bodies, have been shown by M. Biot to depend on a series of thicknesses proportional to those in Newton's scale for thin plates; so with respect to heated glass, Dr. Brewster observes, that a corresponding arithmetical progression is observable for the same tints, whether the thicknesses compared be those of single plates, or the aggregate thickness of several combined.

From these phenomena, Dr. Brewster infers the production of what he calls a crystalline structure in the glass during its contact with heated iron; but observing the existence of an opposite structure in the middle of the glass, and again the same structure at its remote extremity apparently beyond the reach of sensible heat, he says there is nothing analogous but in the perplexing phenomena of magnetic and electric polarity.

In the prosecution of these experiments, the author varies indefinitely the forms and dimensions of his plates of glass, and with them the forms of the fringes produced; but it would be next to impossible to convey any correct idea of the various appearances without assistance from drawings; neither indeed could Dr. Brewster himself have observed the phenomena with sufficient precision in their fluctuating state as arising from the temporary communication of heat, had he not found means to render the same properties permanent,

by a method which forms the subject of the second section of this paper.

When a plate of glass is brought to a full red heat, and is then cooled by placing its edge on a bar of cold iron, the same fringes of colour are developed during cooling as by placing cold glass upon hot iron; and in this case the glass retains the property given to it even after it is completely cold.

The author delineates various configurations of colours produced by plates of different forms thus cooled. Among many others, a parallelogram of glass exhibits an inscribed parallelogram, with lines from each angle to the angles of the plate; and when the plate has been divided longitudinally by a diamond, each of the portions again exhibits an inscribed parallelogram, just as if the parts had been separately heated; and in this respect they present a property analogous to that of a divided magnet, each part of which has opposite poles as the entire bar.

A circular plate of glass, cooled with its centre resting on a piece of cold iron, or a cylindrical rod of glass cooled in the open air, when examined by polarized light in the direction of its axis, each present the same appearance of a black cross through their centre, and concentric fringes parallel to their circumference.

Since it is obvious that in these cases of rapid cooling, as well as those of rapidly heating, there must be progressive variations of density of the glass proceeding in a direction from the source of heat or of cold, and since the phenomena exhibited by many crystallized bodies, when examined in the direction of their axis, are precisely similar, Dr. Brewster infers that there exists in these crystallized bodies also a corresponding variation of density, proceeding toward their axes, which will afford an easy explanation of the fringes they exhibit.

Farther Experiments on the Combustion of explosive Mixtures confined by Wire-gauze, with some Observations on Flame. By Sir Humphry Davy, *LL.D. F.R.S. V.P.R.I.* Read January 25, 1816. [*Phil. Trans.* 1816, *p.* 115.]

In these experiments, the author examines what magnitude of wire and of apertures in the metallic gauze of his lamp is consistent with security against explosion of mixtures externally.

When the gauze is made of wire one fiftieth of an inch in diameter, and at intervals of one tenth, so as to make 100 apertures in the square inch, explosion may take place, either from intense ignition of the top of the lamp, or from lateral currents of air forcing the flame through the interstices.

When the intervals of the same wire were only one fourteenth, though the danger from lateral motion was obviated, still ignition of the wire caused explosion. With intervals of one sixteenth, still there was danger from the same source; but when the distances were reduced to one twenty-fourth on 576 apertures in the square